Locally Differentially Private Frequency Estimation via Joint Randomized Response

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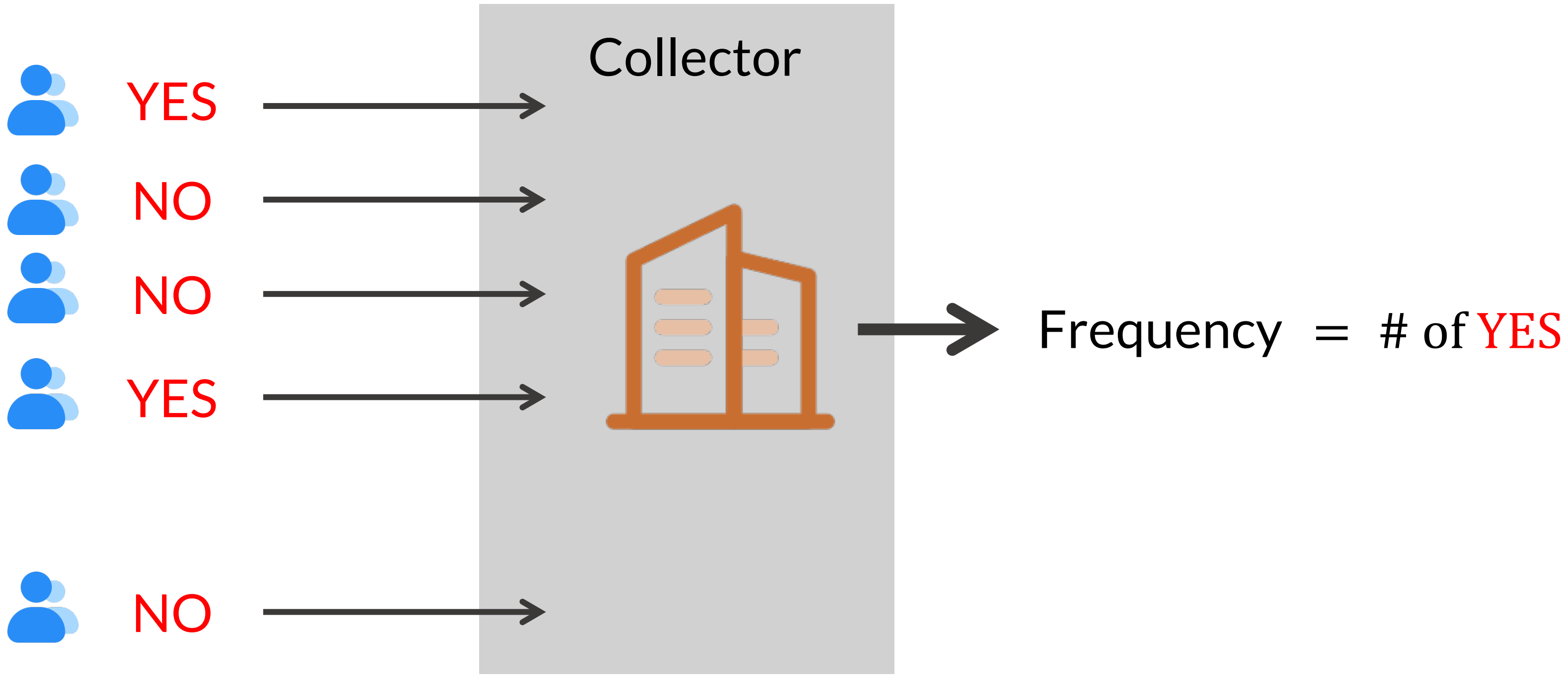
Frequency Estimation

- Social scientists: <u>How many people engage in tax evasion?</u>

  - ask one person if they had evaded tax

  - the person answers YES or NO



Randomized Response for Privacy

- People have privacy concerns on sensitive/embarrassing question

  - i.e. don't want to let the collector know

- A privacy mechanism $\mathcal{M}$ satisfies LDP if

For any truth x_1, x_2,
and randomized answer y:

$$\max \boxed{\frac{\Pr[\mathcal{M}(x_1) = y]}{\Pr[\mathcal{M}(x_2) = y]}} \leq e^{\varepsilon}$$

Distinguishability of x_1 (YES) and x_2 (NO)
from y (randomized answer)

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Distinguishability of x_1 (YES) and x_2 (NO)
from y (randomized answer)

- **quantifiable hardness** to distinguish x_1 (YES) and x_2 (NO) from the randomized answer y

- against inference from data collectors 🏢 or adversaries 👹

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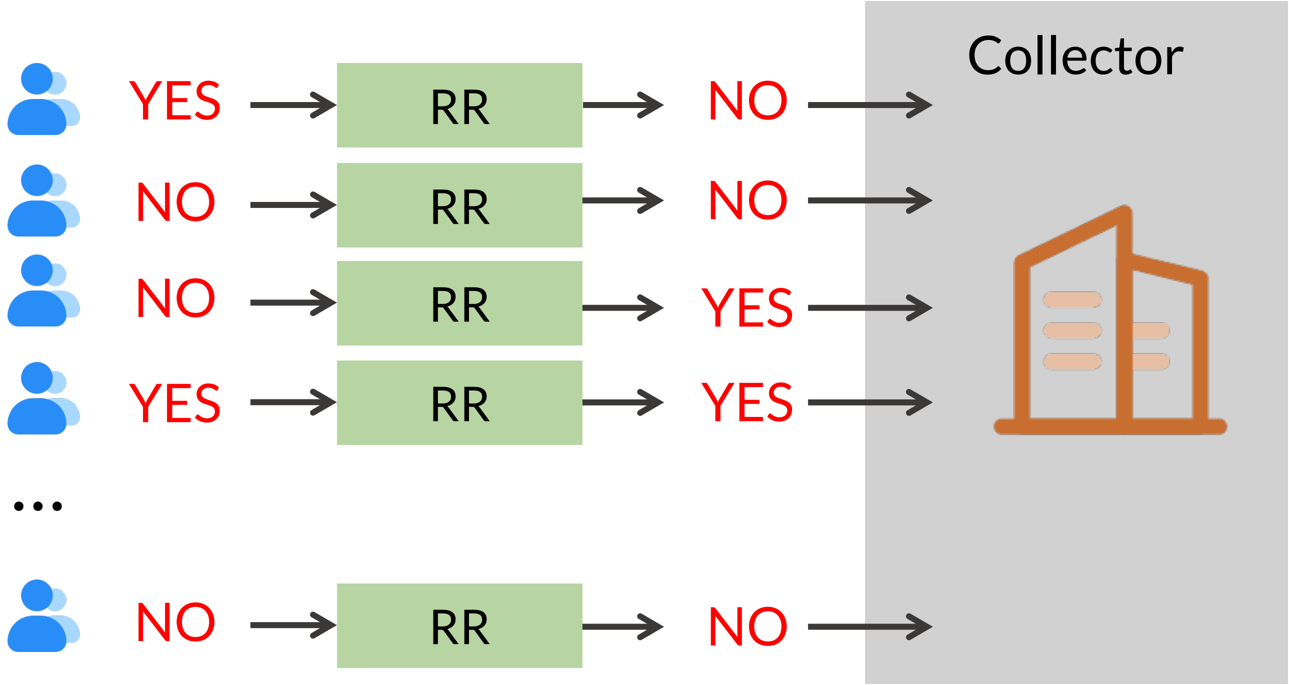
- Randomized Response: Randomize the truth before answering the collector



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$$\max \frac{\Pr[\mathbf{RR}(x_1) = y]}{\Pr[\mathbf{RR}(x_2) = y]} \leq e^{\ln\frac{p}{1-p}}$$

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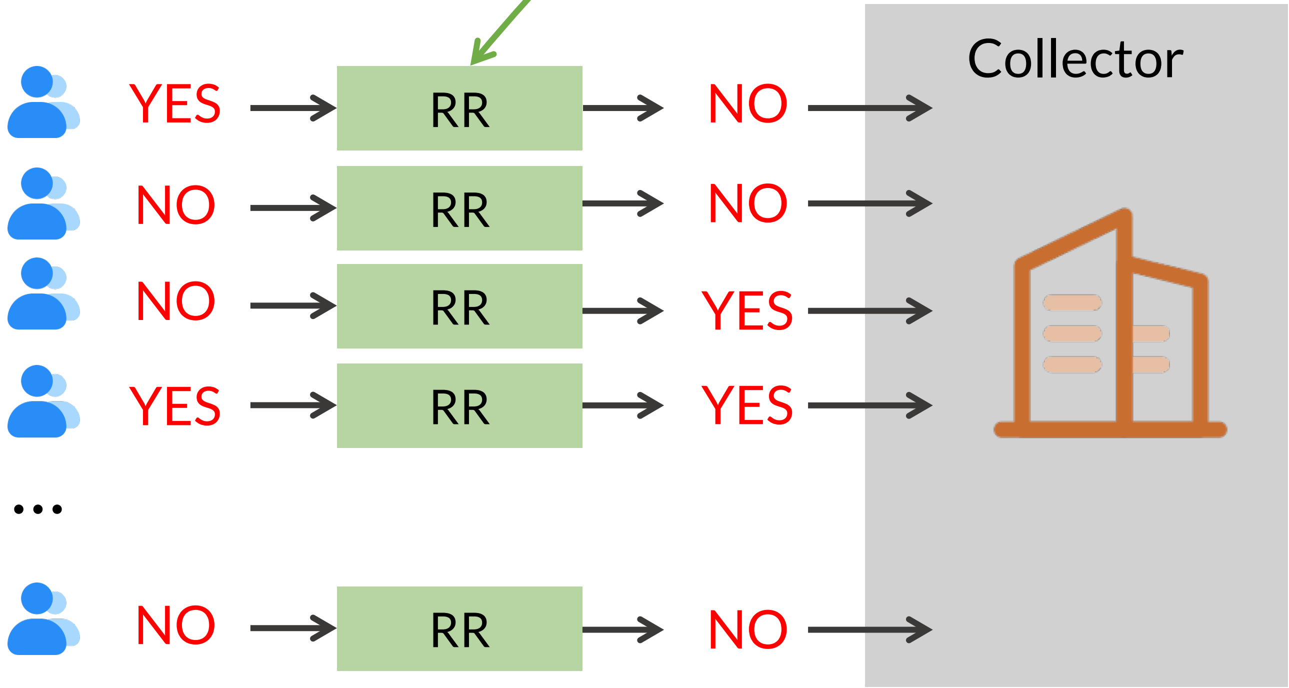
Private

RR: [Warner, 1965]
answer truth with probability p

$$\mathrm{RR}(x) = \begin{cases} x & \text{w.p. } p \\ \neg x & \text{w.p. } 1-p \end{cases}$$



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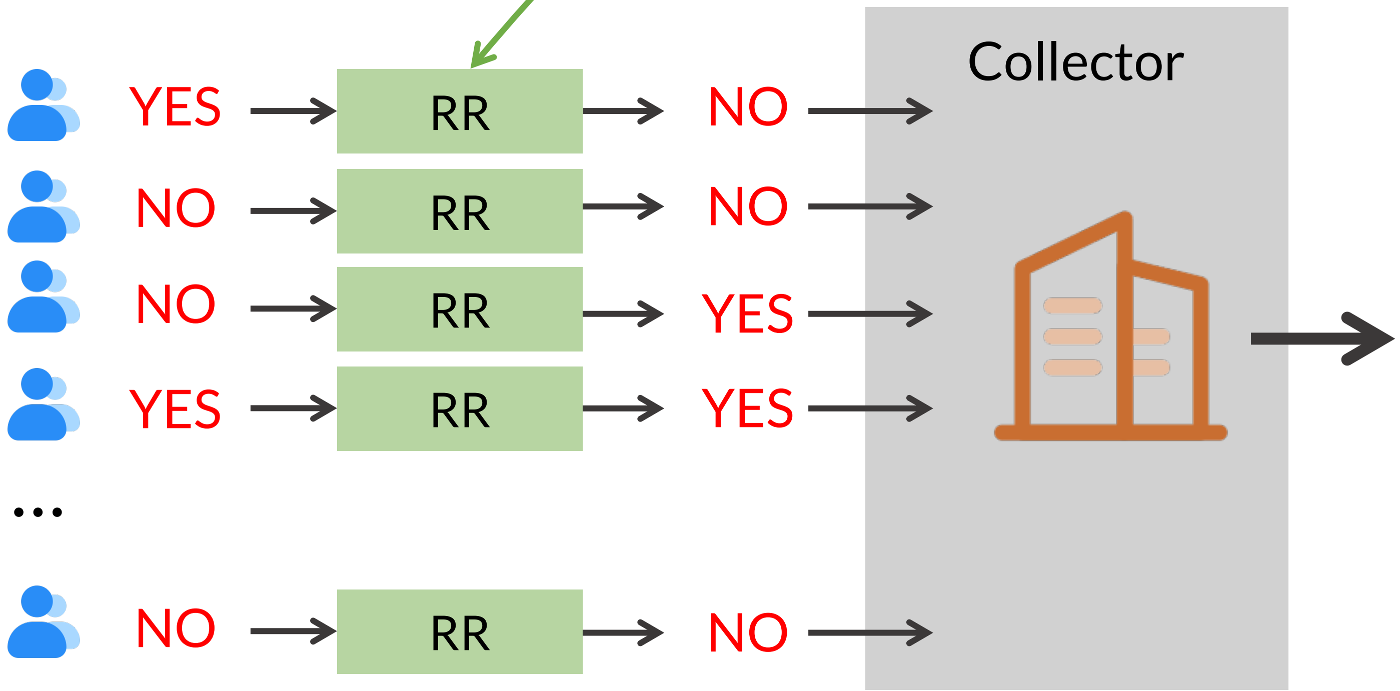

estimated frequency

$$= \frac{\#\text{ of YES} - \# \times q}{p-q}$$

Unbiased:
expectation = truth

Utility: RR's Variance

- Randomization reduces data utility

$$\mathrm{Var}\left[\frac{\#\text{ of }\text{YES} - \# \times q}{p-q}\right] = \frac{\mathrm{Var}[\#\text{ of YES}]}{(p-q)^2} = \frac{npq}{(p-q)^2}$$

\- summation of variance from n **independent** randomization

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- larger $p \in (0.5, 1]$ → lower variance → larger privacy parameter ε

↑ data utility  ↓ privacy

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- Q: Can we improve this privacy-utility tradeoff?

- **yes, by correlated (joint) randomization**

This Paper: Joint RR (JRR)

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- **Example:** 2-person ($x_1 = \text{YES}$ and $x_2 = \text{YES}$) with $p = 0.8$ ($\mathrm{P}[T = 1] = 0.8$)

RR: Joint distribution

	$T_1 = 1$	$T_1 = 0$
$T_2 = 1$	0.64 ($= p^2$)	0.16 ($= pq$)
$T_2 = 0$	0.16 ($= pq$)	0.04 ($= q^2$)

Truthfulness of x_1

Truthfulness of x_2

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Independent T_1 and T_2 ($\mathbf{P}[T_1 \cap T_2] = \mathbf{P}[T_1] \cdot \mathbf{P}[T_2]$)

Joint probability = Π of marginal probabilities

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Same marginal prob for each person

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$\mathbf{P}[T_1 = 0 \cap T_2 = 0] = \mathbf{0} \neq \mathbf{P}[T_1 = 0] \cdot \mathbf{P}[T_2 = 0] = \mathbf{0.04}$

NOT independent T_1 and T_2

Joint probability $\neq$ Π of marginal probabilities

Utility: JRR's Variance

- Same estimator as RR

$$\mathrm{E}[\#\text{ of YES}] = \sum_{i=1}^{\#} \mathrm{P}[y_i = \text{YES}] = n_{\text{YES}} \cdot p + (\# - n_{\text{YES}}) \cdot q$$

Expectation

Ground truth

Utility: JRR's Variance

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$$\mathrm{E}[\#\text{ of }\mathrm{YES}] = \sum_{i=1}^{\#} \mathrm{P}[y_i = \mathrm{YES}] \;= n_{\mathrm{YES}} \cdot p + (\# - n_{\mathrm{YES}}) \cdot q$$

→ Unbiased estimator $\hat{n}_{\mathrm{YES}} = \boxed{\dfrac{\#\text{ of }\mathrm{YES} - 2q}{p-q}}$

Identical to RR

Utility: JRR’s Variance

- Variance: (# = 2, $p = 0.8$)

$$\mathrm{Var}[\hat{n}_{\mathrm{YES}}] = \frac{\mathrm{Var}[\#\text{ of YES}]}{(0.8-0.2)^2}$$

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YES → RR / JRR → YES / NO

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- Distribution table:

RR

# of YES	0	1	2
Probability	0.04	0.16 + 0.16	0.64

$$\mathrm{Var}[\#\text{ of YES}] = E[(X-\mu)^2] = \mathbf{0.32}$$

$$= \sum_{X=0,1,2} (X-1.6)^2 \cdot \Pr[X] \approx \mathbf{0.1 + 0.12 + 0.1}$$

JRR

# of YES	0	1	2
Probability	0	0.2 + 0.2	0.6

$$\mathrm{Var}[\#\text{ of YES}] = E[(X-\mu)^2] = \mathbf{0.24}$$

$$= \sum_{X=0,1,2} (X-1.6)^2 \cdot \Pr[X] = \mathbf{0 + 0.14 + 0.1}$$

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Better utility

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JRR **(near to μ)**

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JRR's General Form

- Correlated randomization with 2 persons x_{2i-1} and x_{2i}

JRR: Joint distribution

	$T_{2i-1} = 1$	$T_{2i-1} = 0$
$T_{2i} = 1$	$p^2 + \rho pq$	$(1 - \rho)pq$
$T_{2i} = 0$	$(1 - \rho)pq$	$q^2 + \rho pq$

$\rho \in [-1,1]$: correlated coefficient

- RR is a special case of JRR with $\rho = 0$ (no correlation)

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Utility Theorem. The variance of JRR's estimator $\hat{\boldsymbol{n}}_{\boldsymbol{v}}$ is

$$\mathrm{Var}[\hat{\boldsymbol{n}}_{\boldsymbol{v}}] = \frac{pq}{(p-q)^2} \cdot \left(n + \frac{\rho\left((2n_{\mathrm{YES}} - n)^2 - n\right)}{n-1} \right).$$

Privacy: NOT as Simple as RR

- If any person can be an adversary

JRR: Joint distribution

	$T_1 = 1$	$T_1 = 0$
$T_2 = 1$	0.6	0.2
$T_2 = 0$	0.2	0

T_1: I am an adversary 😈

When I report untruthfully $(T_1 = 0)$,
My partner will report truthfully $(T_2 = 1)$

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- Correlation results in privacy leakage

JRR – Privacy Model in This Paper

- Randomly groups into form 2-person groups for correlated randomization

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- **Threat model:**

  - any person can be an adversary 👹

  - if a group contains an adversary, the adversary knows who is their partner (after random grouping)

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A person

Partner is an 👹

Partner is NOT an adversary

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$$\Pr = \frac{\#\text{😈}}{\#\text{👥}}$$

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- Randomly groups into form 2-person groups for correlated randomization

- **Threat model:**

  - if a group contains an adversary, the adversary knows who is their partner (after random grouping)

  - the adversary cannot control randomness, but can infer their partner's

$$\Pr = \frac{\#\text{👿}}{\#\text{👤}}$$

A person

Partner is an 👿

$$\Pr[\text{person} \mid \text{👿}] = \frac{(1-\rho)pq}{p}$$ Higher confidence

Partner is NOT an adversary

JRR – Formal Privacy & Utility

Privacy Theorem. Assume a set of data contributors $\mathcal{T}_m$ whose reporting truthfulness is known to the adversary. For any data contributor i, the JRR mechanism satisfies:

$$\frac{\Pr[\,\mathrm{JRR}(x_i)\mid \mathcal{T}_m]}{\Pr[\,\mathrm{JRR}(x_i')\mid \mathcal{T}_m]} \leq e^{\varepsilon}, \quad \text{where} \quad \varepsilon = \ln \frac{mp_{\max} + (n-m-1)p}{mp_{\min} + (n-m-1)q}.$$

$p_{\max} = \max\{(1-\rho)p, p + \rho q\}$: confidence of adversaries inferring a specific value

Privacy affected by

m	# adversaries 👹
n	# of persons 👥
ρ	Correlated coefficient

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privacy constraint

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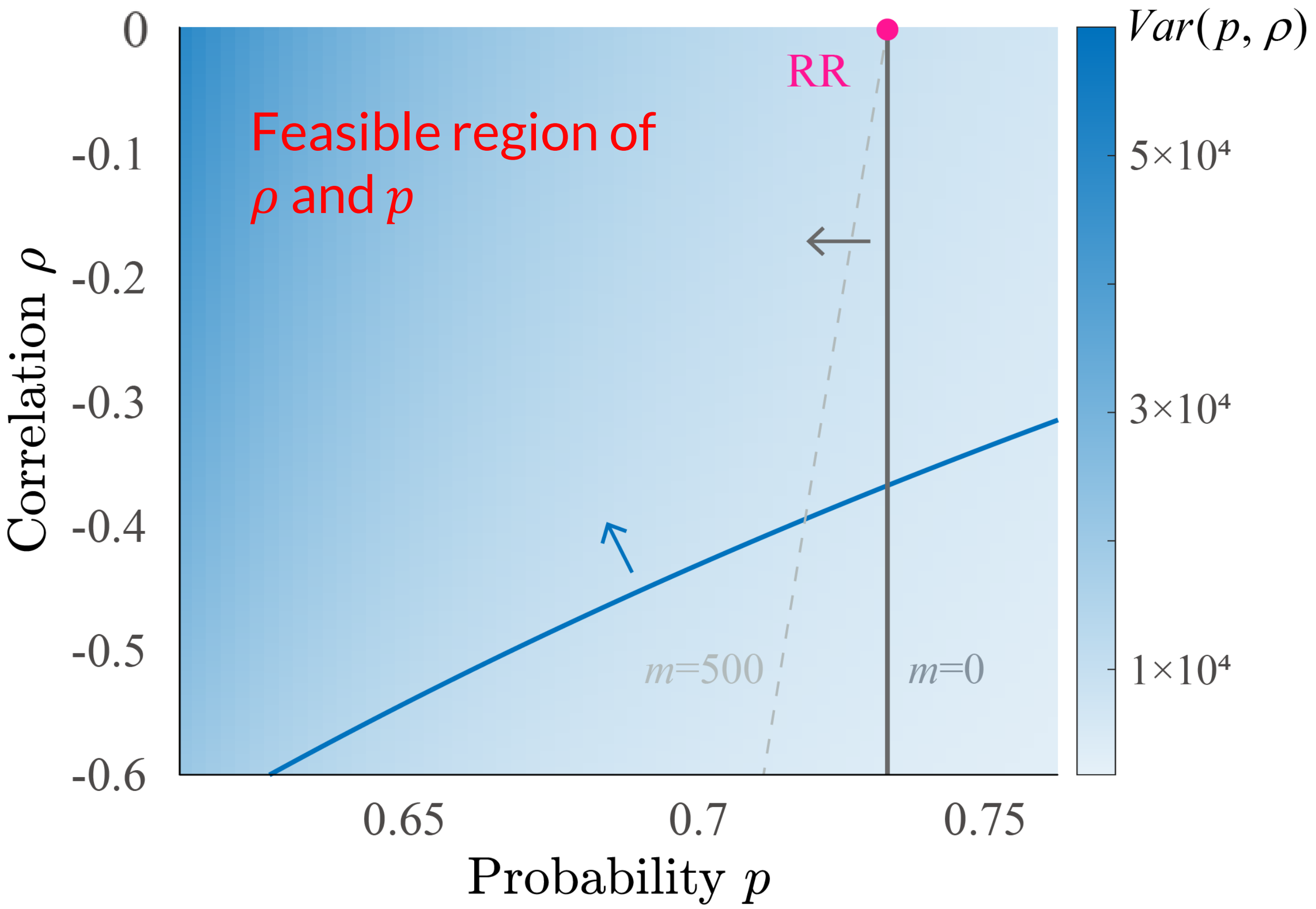
minimize

$$\mathrm{Var}[\hat{\boldsymbol{n}}_v] = \frac{pq}{(p-q)^2} \cdot \left(n + \frac{\rho\left((2n_{\mathrm{YES}} - n)^2 - n\right)}{n - 1} \right).$$

JRR – Variance Heatmap

- Effect of ρ and p (when $\varepsilon = 1, n = 10^4, n_{\text{Yes}} = 200$, and $m = 0$ & 500)



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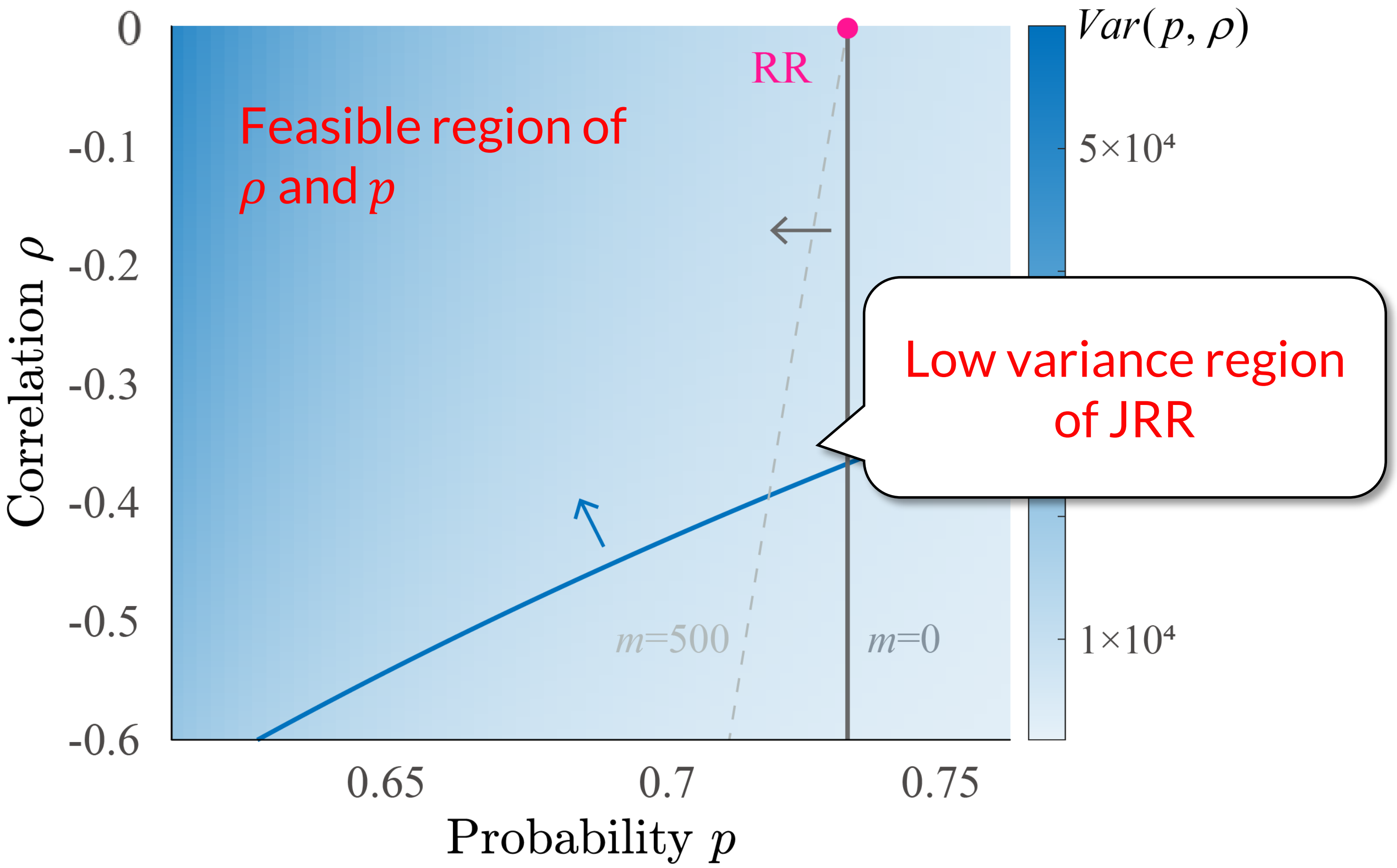
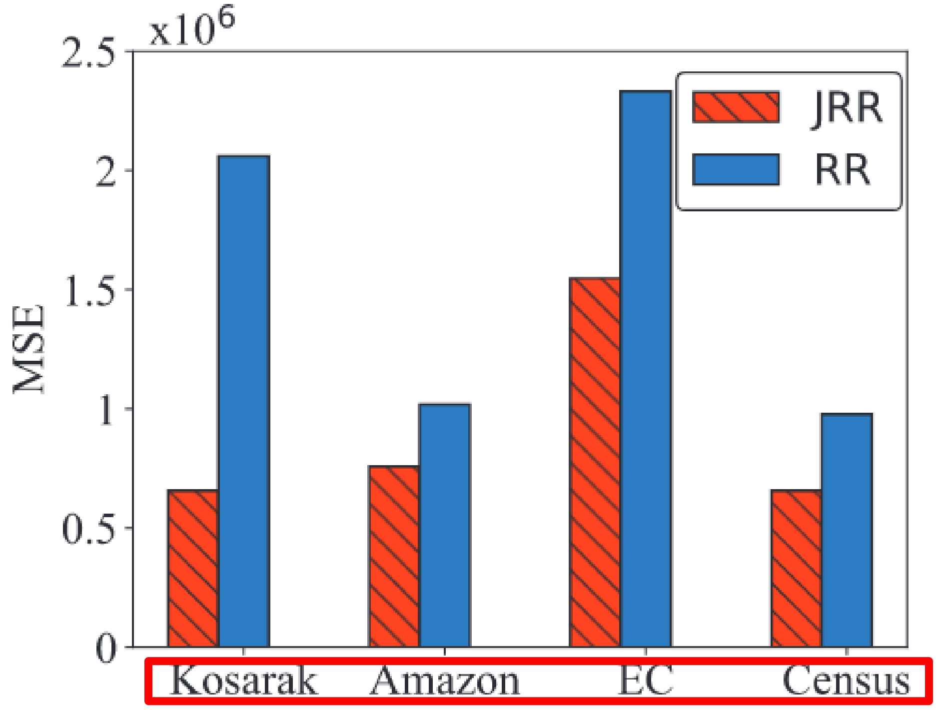
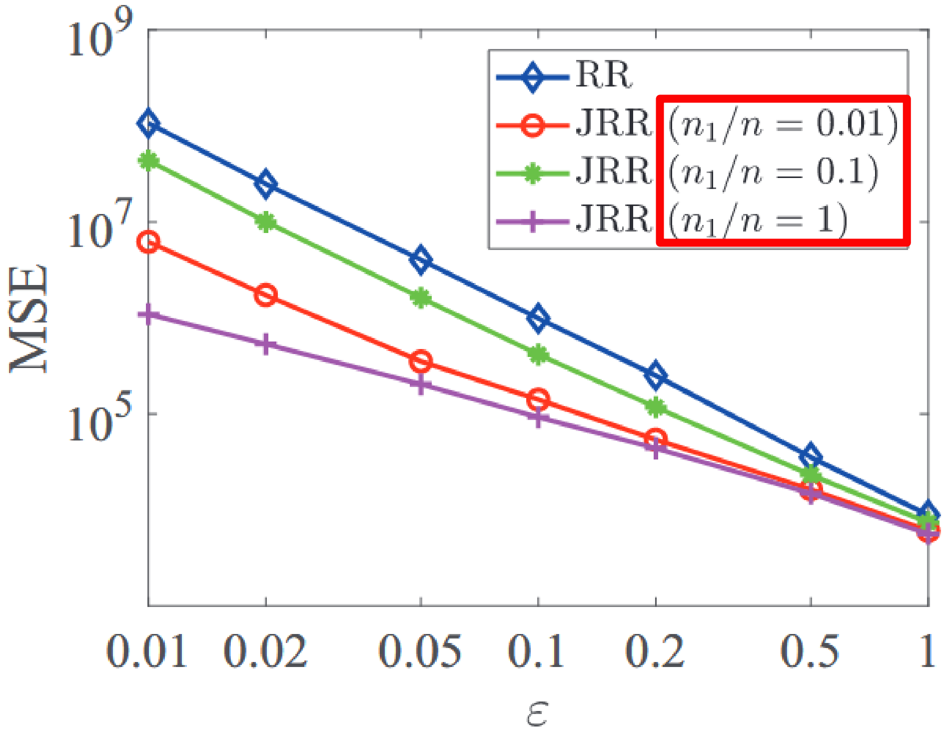


Experiments

- Comparison with RR under the same privacy level - JRR: $\varepsilon(n, m, \rho, p)$, RR: $\varepsilon(p)$



Real-world datasets ($\varepsilon = 0.1$)



Synthetic datasets ($n = 10^4$)

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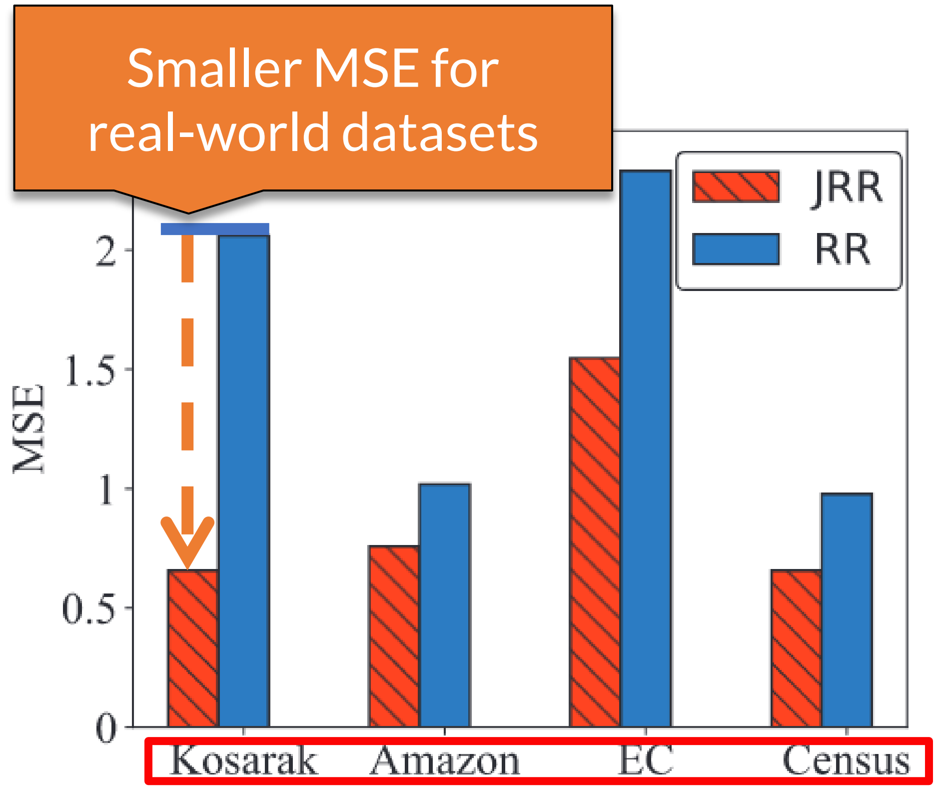

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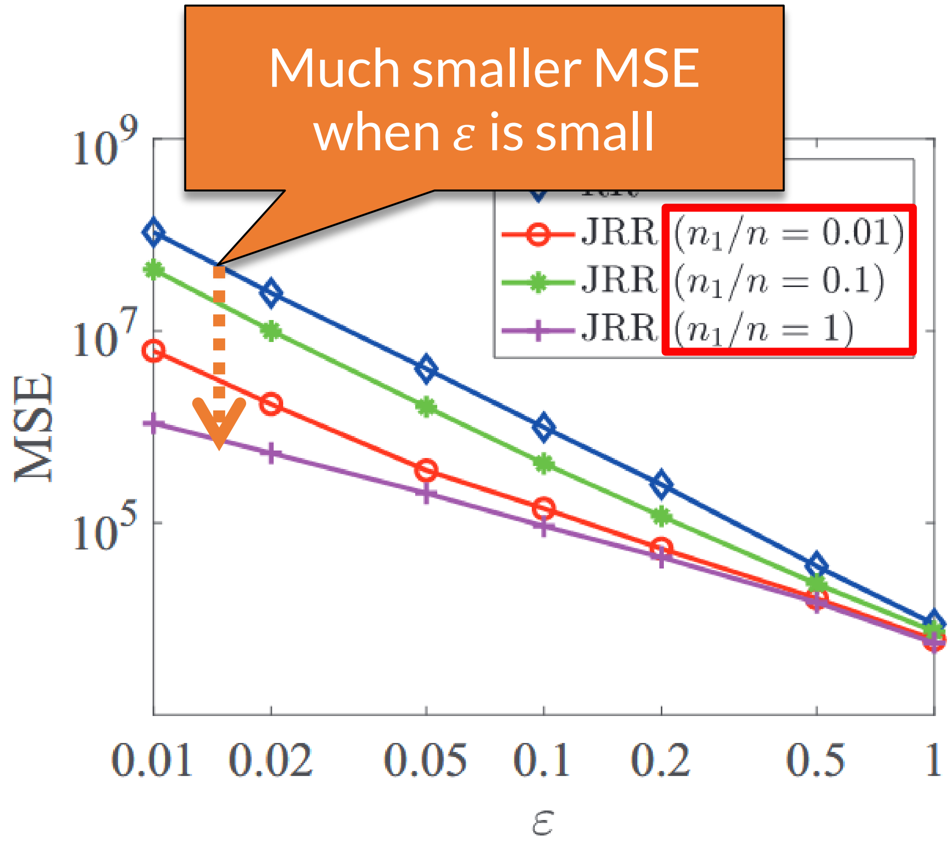

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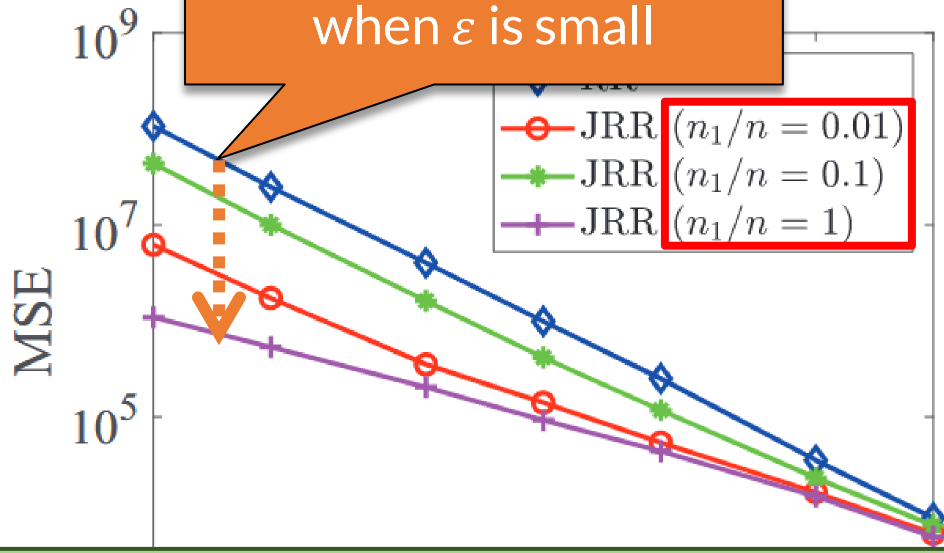
Smaller MSE for real-world datasets

Much smaller MSE when ε is small



- Correlated randomization can improve the data utility of frequency estimation
- **JRR: Privacy & utility model for correlated randomization**

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Randomized Response for Privacy

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  - i.e. don't want to let the collector know
- Randomized Response: Randomize the truth before answering

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answer truth with probability p

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YES →	RR	→ NO →	Collector
NO →	RR	→ NO →	
NO →	RR	→ YES →	
YES →	RR	→ YES →	
…			
NO →	RR	→ NO →	

estimated frequency

$= \frac{\#\text{ of YES} - \# \times q}{p - q}$

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expectation = truth

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$\mathrm{P}[T_1 = 0 \cap T_2 = 0] = \mathbf{0} \neq \mathrm{P}[T_1 = 0] \cdot \mathrm{P}[T_2 = 0] = 0.04$

NOT independent T_1 and T_2

Joint probability ≠ Π of marginal probabilities

Utility: JRR's Variance

- Variance: (# = 2, $p = 0.8$)

$\mathrm{Var}[\hat{n}_{\mathrm{YES}}] = \frac{\mathrm{Var}[\#\text{ of YES}]}{(0.8-0.2)^2}$

Randomized

YES → RR / JRR → YES / NO
YES → YES / NO

- Distribution table:

Better utility

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JRR (near to μ)

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- Correlated randomization with 2 persons x_{2i-1} and x_{2i}

JRR: Joint distribution

	$T_{2i-1} = 1$	$T_{2i-1} = 0$
$T_{2i} = 1$	$p^2 + \rho pq$	$(1-\rho)pq$
$T_{2i} = 0$	$(1-\rho)pq$	$q^2 + \rho pq$

$\rho \in [-1,1]$: correlated coefficient

- RR is a special case of JRR with $\rho = 0$ (no correlation)

JRR – Privacy Model in This Paper

- Randomly groups into form 2-person groups for correlated randomization
- **Threat model:**
  - if a group contains an adversary, the adversary knows who is their partner (after random grouping)
  - the adversary cannot control randomness, but can infer their partner's

$\Pr = \frac{\#}{\#}$

A person

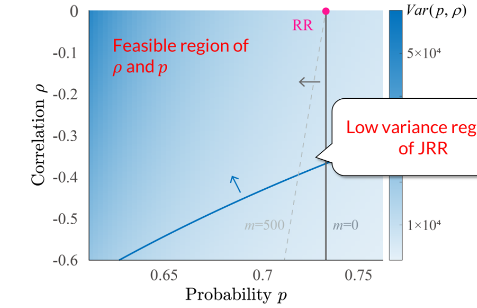
Partner is an

Partner is NOT an adversary

→ $\Pr[\text{person} \mid \ ] = \frac{(1-\rho)pq}{p}$ Higher confidence

JRR – Variance Heatmap

- Effect of ρ and p (when $\varepsilon = 1, n = 10^4, n_{\mathrm{Yes}} = 200$, and $m = 0$ & 500)



Thank you!

Q&A

Privacy Model

- No need for securing shuffling:

  - when one person hold multiple items