TraCS: Trajectory Collection in Continuous Space under Local Differential Privacy

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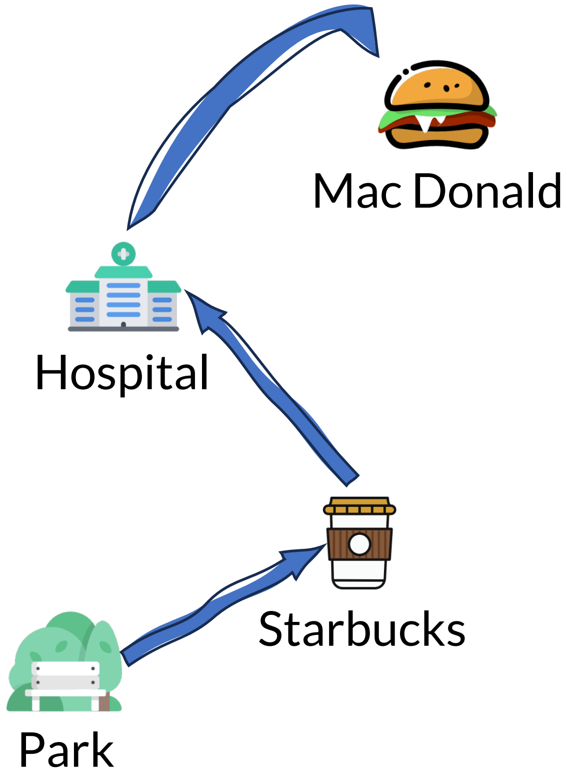
Rochester Institute of Technology (RIT)

Trajectory Collection

- Private trajectories

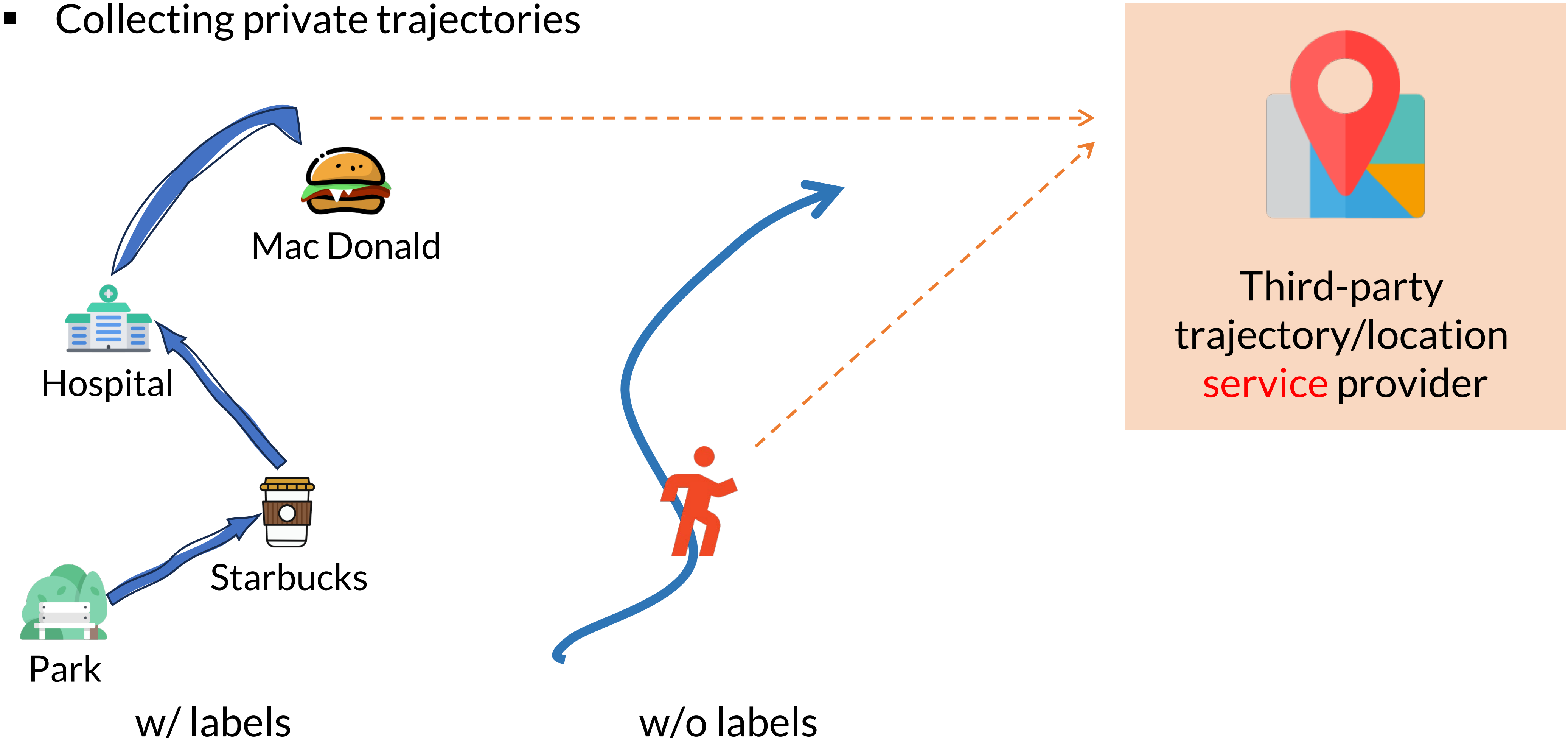


w/ labels

w/o labels

Trajectory Collection

- Collecting private trajectories



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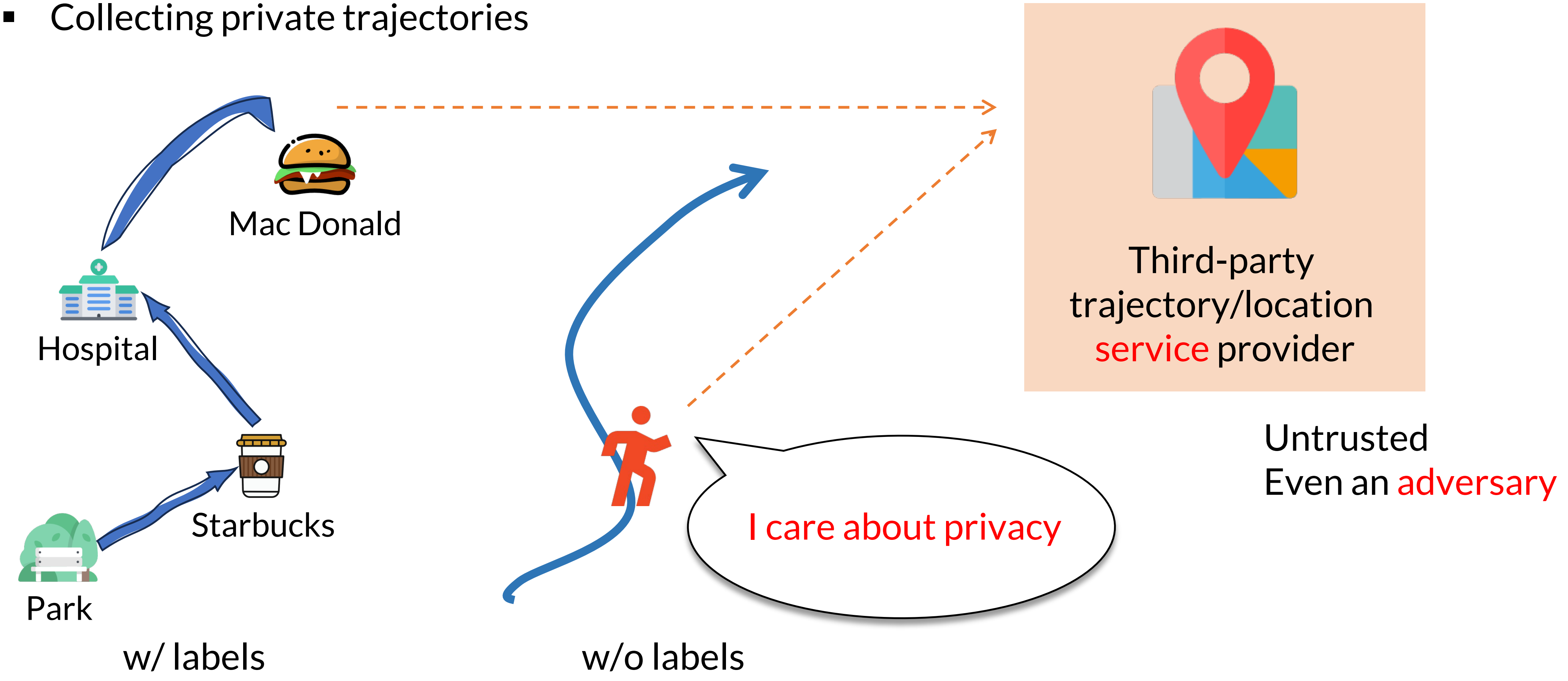

LDP-fy a Trajectory

- LDP-fy: perturb a trajectory with LDP guarantee (provable privacy)

  - cannot distinguish location τ_1 from τ_2 with confidence level e^{ε}

$$\forall \tau_1, \tau_2, \tau_* \in \mathcal{S}: \quad \frac{\Pr[\mathcal{M}(\tau_1) = \tau_*]}{\Pr[\mathcal{M}(\tau_2) = \tau_*]} \le e^{\varepsilon}$$

distinguish level

location space

probability difference of outputting τ_*

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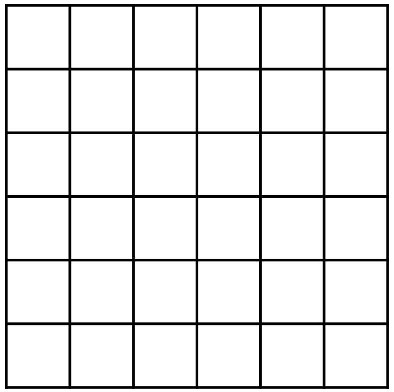
- **We want:** $\mathcal{M}$ for continuous location space

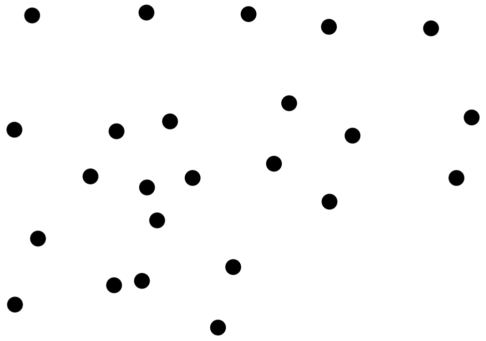
  - existing methods are all for discrete location space → not always available
  - continuous space ⊃ discrete space → can apply to discrete space

Existing Methods

- Location space $\mathcal{S}$ is a set of cells or points



$\mathcal{S} = \{c_1, \dots, c_n\}$



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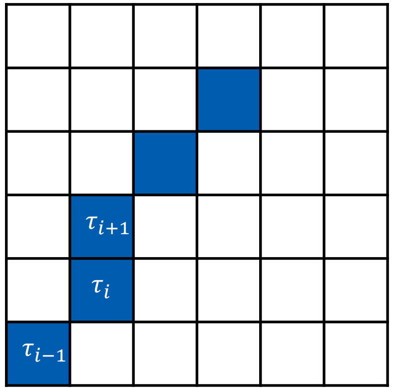

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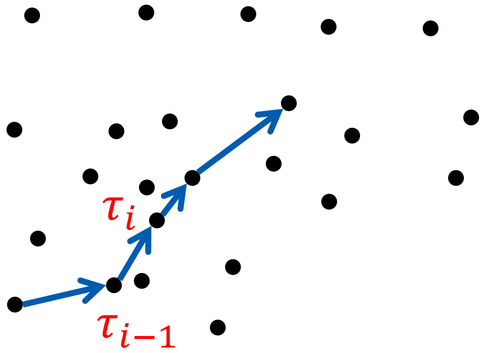

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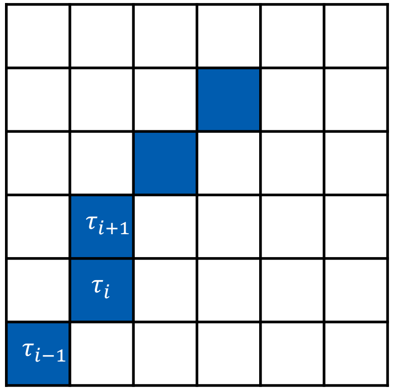

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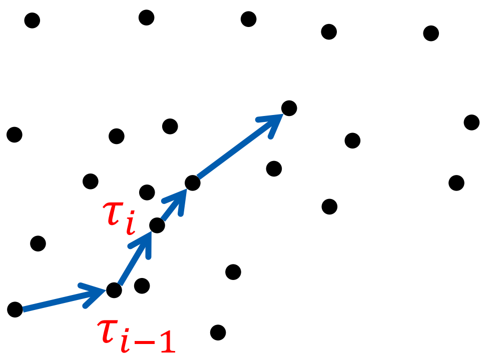

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- Perturb each location τ using the Exponential Mechanism

$$\Pr[\mathcal{M}_{\exp}(\tau) = \tau^*] = \frac{\exp\left(kd(\tau, \tau^*)\right)}{\sum_{\tau' \in S} \exp\left(kd(\tau, \tau')\right)}$$

distance

sum of distance

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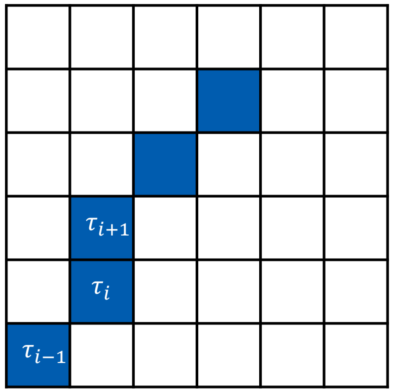

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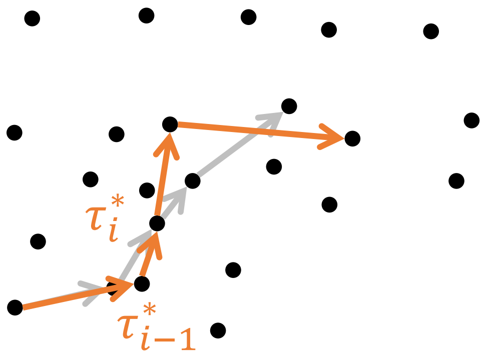

Perturbed trajectory (ensuring LDP)

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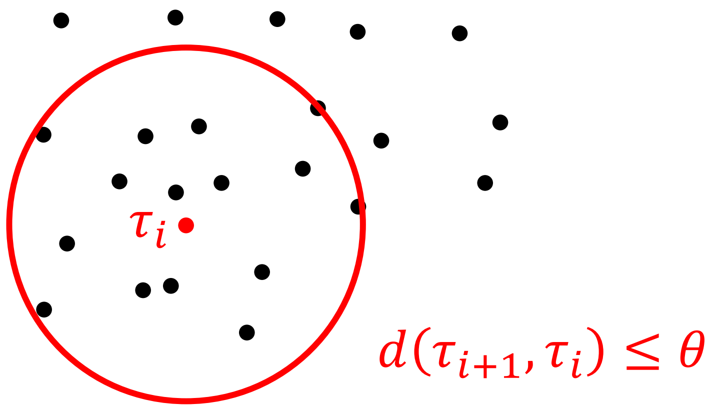
- NGram [2021]: reachability constraint from public knowledge

  - e.g. distance reachability

  - the next location cannot be too far

  - $\mathcal{M}_{\exp}(\tau)$ on reduced $\mathcal{S}$



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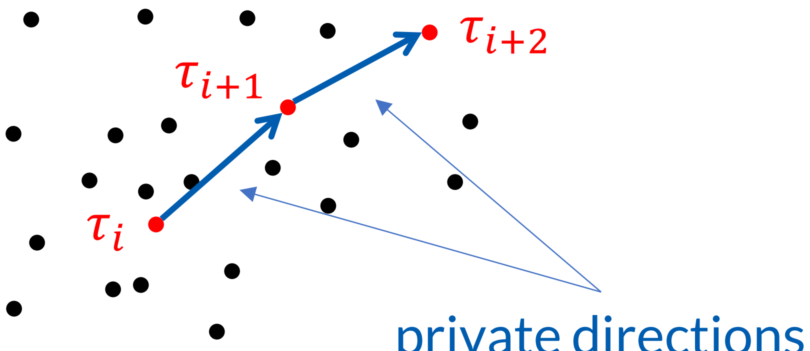

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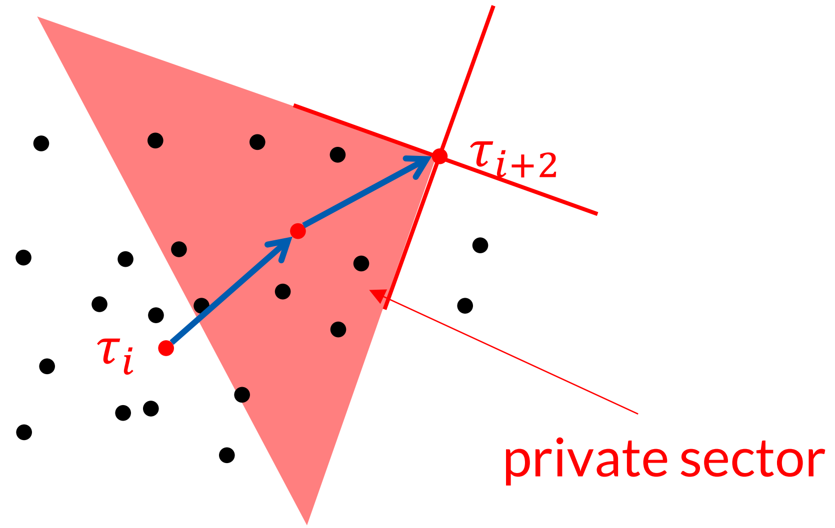

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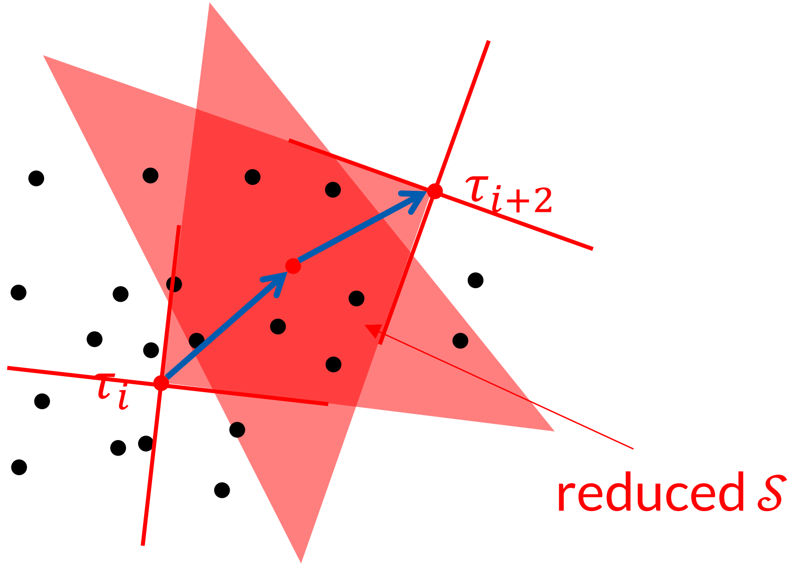
  - *i.* divide direction sectors, e.g. $k = 4$

  - *ii.* perturb sector

  - *iii.* $\mathcal{M}_{\text{exp}}(\tau)$ on reduced $\mathcal{S}$



Existing Methods: **Weaknesses**

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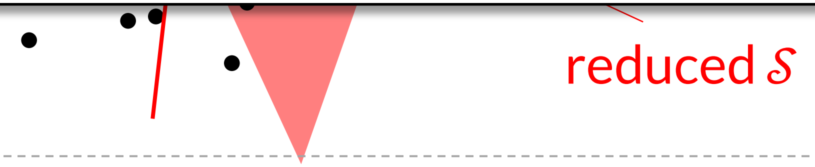
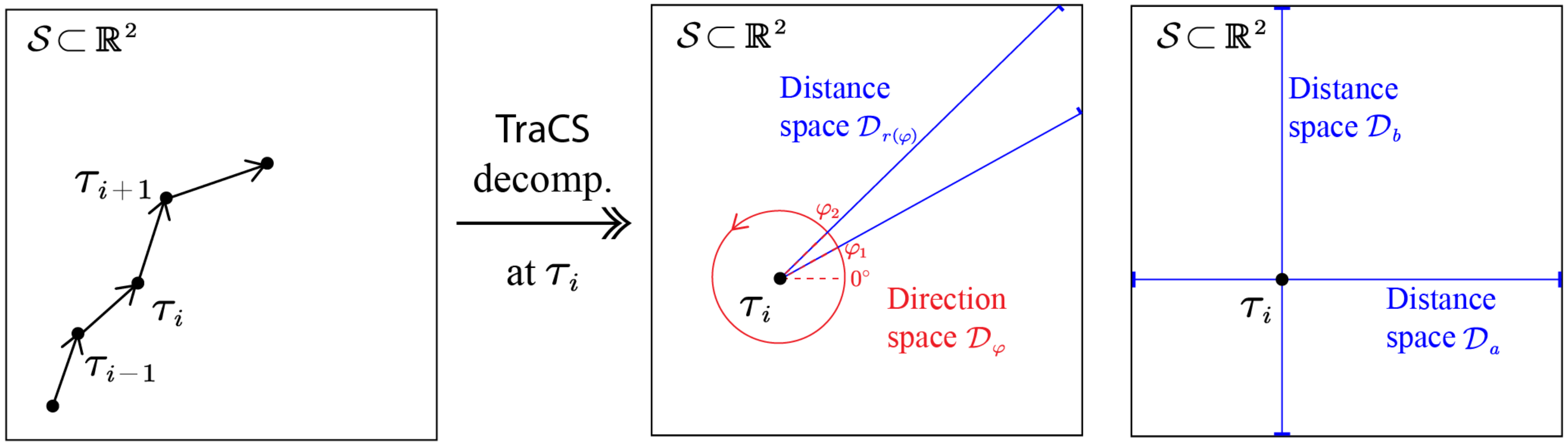


This Paper: Continuous Space

- TraCS-D: direction-distance perturbation
- TraCS-C: coordinates perturbation

- **Key idea:** decomposes $\mathcal{S}$ into two subspaces



TraCS-D: $\mathcal{D}_\varphi \otimes \mathcal{D}_{r(\varphi)}$ TraCS-C: $\mathcal{D}_a \times \mathcal{D}_b$

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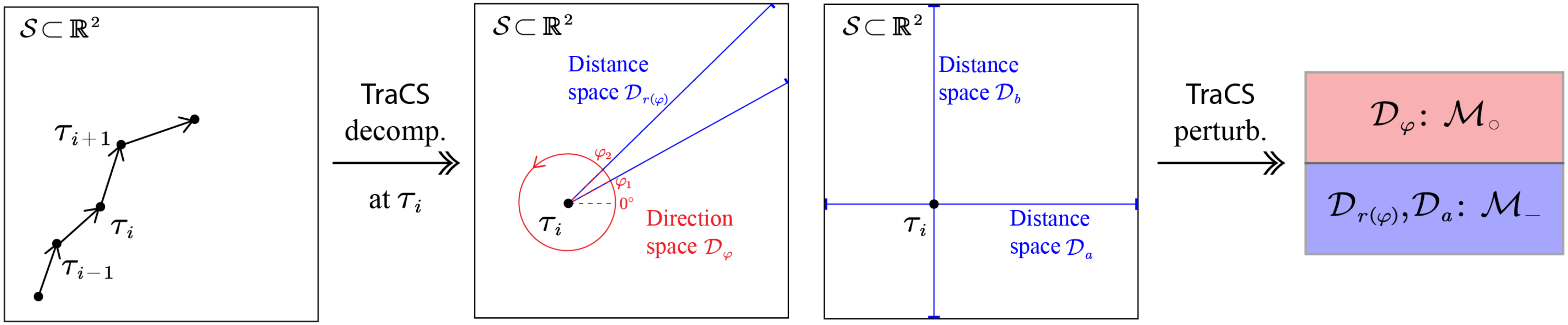


TraCS-D: $\mathcal{D}_\varphi \otimes \mathcal{D}_{r(\varphi)}$

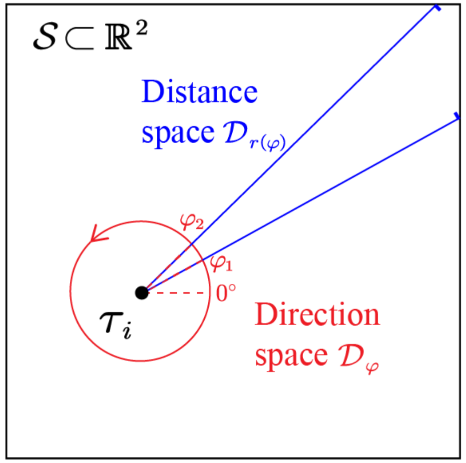
TraCS-C: $\mathcal{D}_a \times \mathcal{D}_b$

Decomposition of Continuous Space

- $\mathcal{S} = \mathcal{D}_{\varphi} \otimes \mathcal{D}_{r(\varphi)}$

2D space   1D subspaces

- Each location $\tau_{i+1} \in \mathcal{S}$ has a unique representation $(\varphi, r(\varphi))$
- Perturb φ and $r(\varphi)$ using 1D mechanisms



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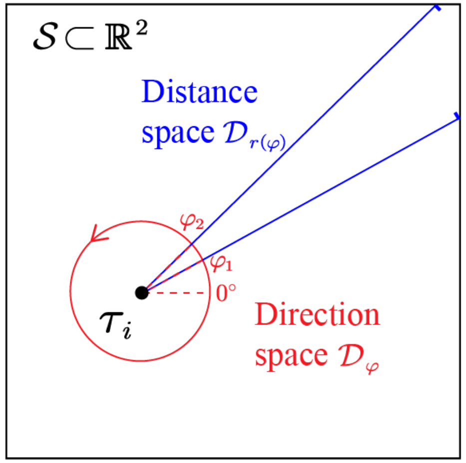

- $\mathcal{S} = \mathcal{D}_a \times \mathcal{D}_b$
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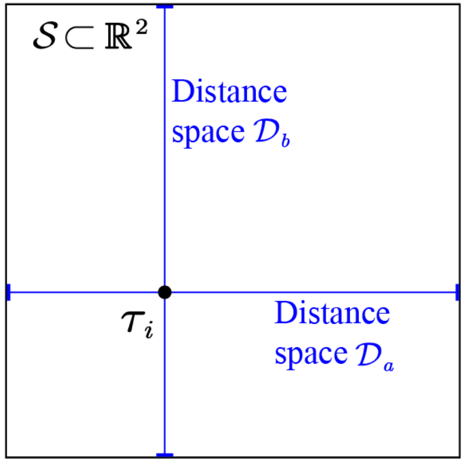
- Perturb a and b using 1D mechanisms



TraCS-C: $\mathcal{D}_a \times \mathcal{D}_b$

$\mathcal{M}$ for Continuous Space

- **Q:** How to design LDP mechanisms for:

  \- circular space $[0,2\pi) \to [0,2\pi)$? linear space $[a_{\text{sta}}, a_{\text{end}}) \to [a_{\text{sta}}, a_{\text{end}})$?

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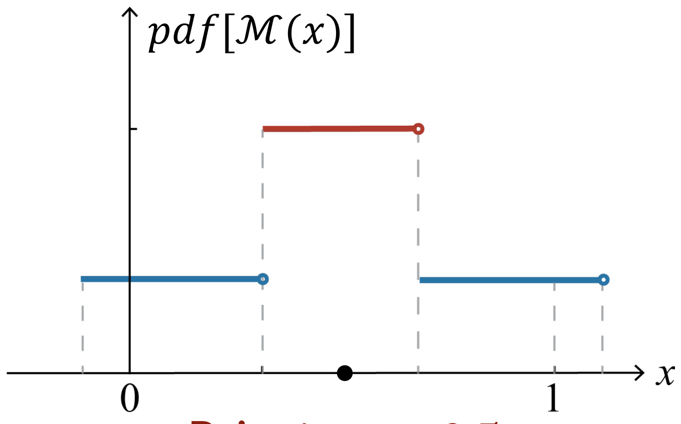
- **S:** Piecewise-based mechanism

  - originally designed for mean * /distribution estimation**

$$pdf[\mathcal{M}(x) = y] = \begin{cases} p_\varepsilon & \text{if } y \in [l_{x,\varepsilon}, r_{x,\varepsilon}), \\ p_\varepsilon / \exp(\varepsilon) & \text{otherwise,} \end{cases}$$

  - ensure LDP for $[0,1) \to [-C, C)$



Private $x = 0.5$

* Collecting and Analyzing Multidimensional Data with Local Differential Privacy, ICDE 2019

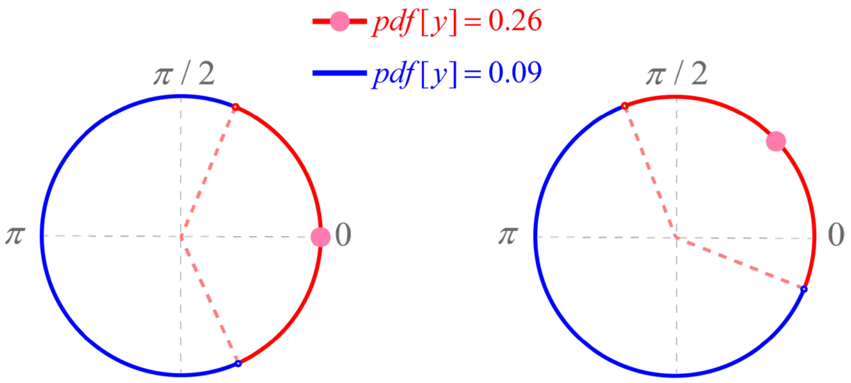
** Estimating Numerical Distributions under Local Differential Privacy, SIGMOD 2020

$\mathcal{M}_\circ$ and $\mathcal{M}_-$

- Design piecewise-based mechanisms $\mathcal{M}_\circ$ and $\mathcal{M}_-$

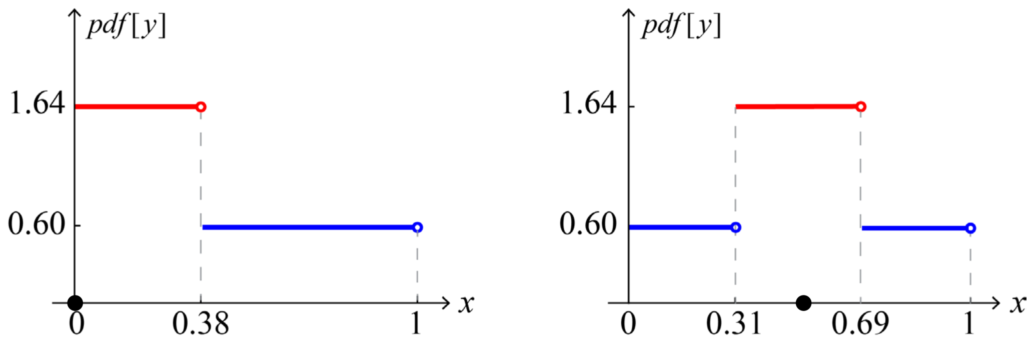
$[0,2\pi) \rightarrow [0,2\pi)$ $[0,1) \rightarrow [0,1)$

- Examples of $\mathcal{M}_\circ$ and $\mathcal{M}_-$



(a) $x = 0$ (b) $x = \pi / 4$

$\mathcal{M}_\circ(x; \varepsilon = 1)$



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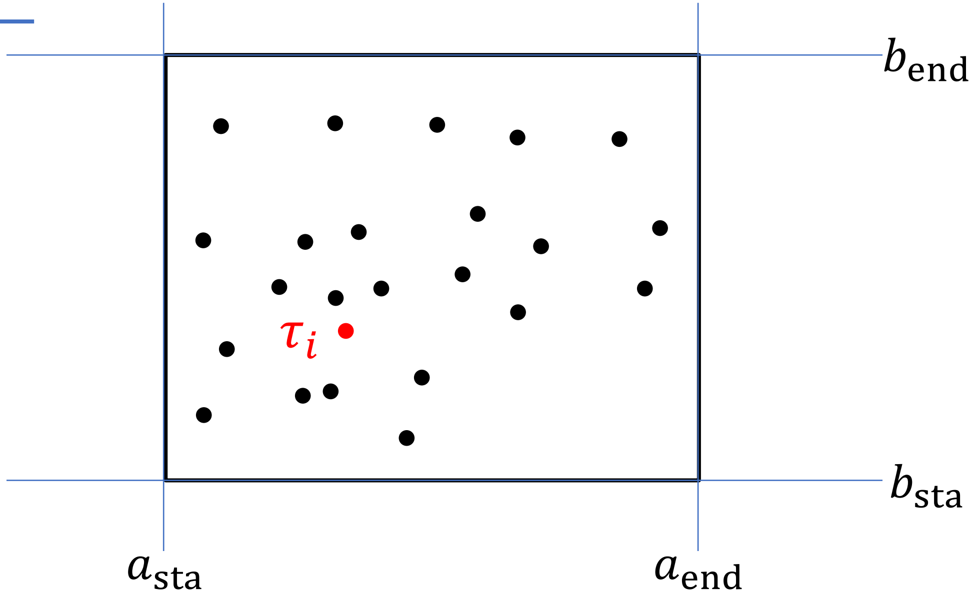
$\mathcal{M}_-(x; \varepsilon = 1)$

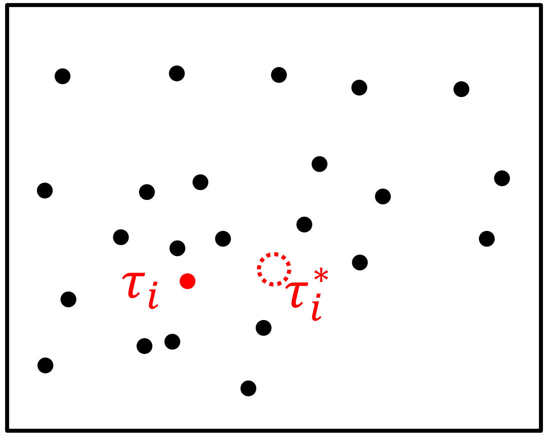
Rounding to Discrete Spaces

- $\underbrace{\mathcal{S} = [a_{\text{sta}}, a_{\text{end}}]}_{\text{longitude}} \times \underbrace{[b_{\text{sta}}, b_{\text{end}}]}_{\text{latitude}}$



Rounding to Discrete Spaces

- $\mathcal{S} = \underbrace{[a_{\mathrm{sta}}, a_{\mathrm{end}}]}_{\text{longitude}} \times \underbrace{[b_{\mathrm{sta}}, b_{\mathrm{end}}]}_{\text{latitude}} \rightarrow$ apply TraCS to $\mathcal{S}$



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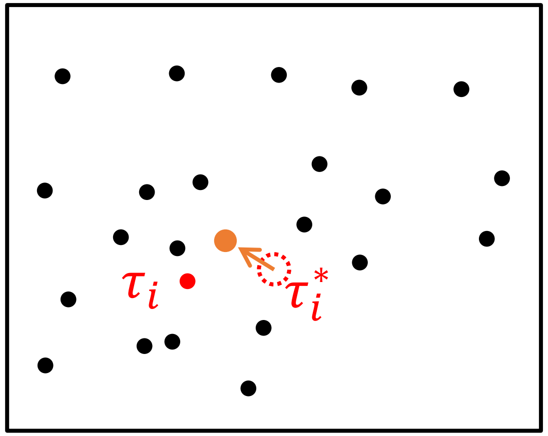

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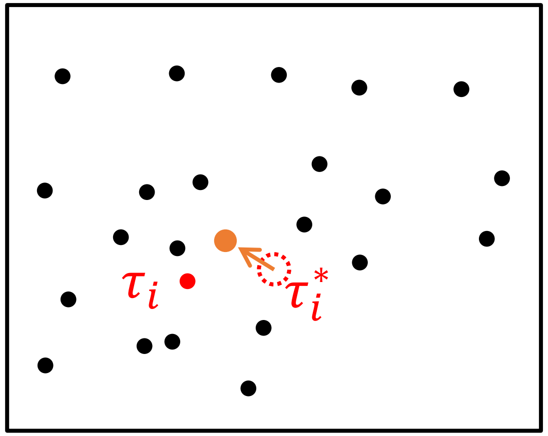

- **Advantages:**

  - not affected by # locations/cells

  - efficient sampling ($\Theta(1)$ time complexity; EM: $\Theta(m)$)

  - stronger privacy (for continuous space)

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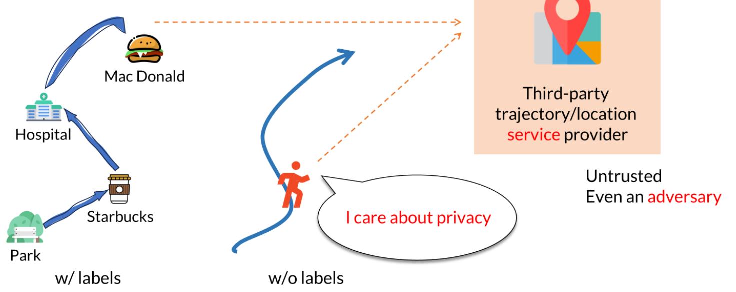

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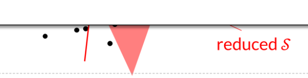

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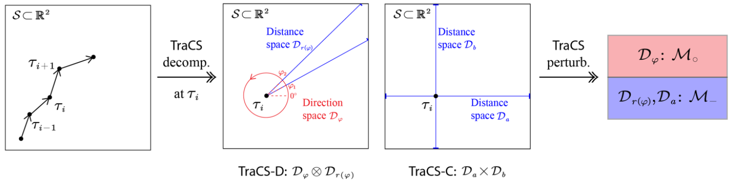

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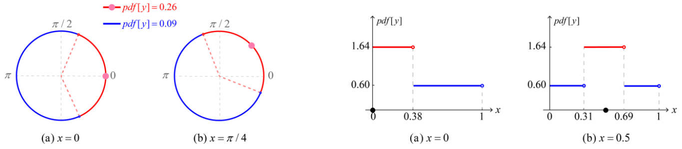

$\mathcal{M}_{\circ}(x; \varepsilon = 1)$ $\quad \mathcal{M}_{-}(x; \varepsilon = 1)$

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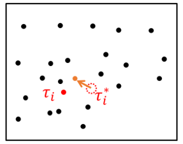

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Q & A